nAMIBIA UnIVERSITY
OF SCIEnCE AnD TECHnOLOGY
FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 35BHAM	**LEVEL:** 8
COURSE CODE: AOR802S	**COURSE NAME:** APPLIED OPERATIONS RESEARCH
SESSION: NOVEMBER 2019	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	PROF. S. A. REJU
MODERATOR:	PROF. O. D. MAKINDE

INSTRUCTIONS
1. Attempt ALL the questions.
2. All written work must be done in blue or black ink and sketches must be done in pencil.
3. Use of COMMA is not allowed as a DECIMAL POINT.
4. Marks will not be awarded for answers obtained without showing the necessary steps leading to them (the answers).

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 4 PAGES (including this front page)

QUESTION 1 [30 MARKS]

(a) Discuss the Reduction by Dominance procedure and hence simplify by using reduction by dominance the game defined by the following payoff matrix, showing progressively the reduced pay-off matrix:

B

$$
A \quad
\begin{array}{c}
\\
A \\
B \\
C \\
D
\end{array}
\begin{array}{ccc}
a & b & c \\
\left[\begin{array}{ccc}
1 & -1 & -5 \\
4 & -4 & 2 \\
3 & -3 & -10 \\
5 & -5 & -4
\end{array}\right]
\end{array}
$$

(8 Marks)

(b) Distinguish between pure and mixed strategies.   (5 Marks)

(c) Eugene has a 250-gallon capacity home heating oil tank, presently empty, meant to store oil against the next winter. Consider the following winter heating oil quantity needed and the oil prices during probable four levels of winter severity:

WINTER SEVERITY	OIL STORAGE NEEDED	OIL PRICES PER GALLON
Mild Winter (MW)	110 Gallons	N$1.00
Average Winter (AW)	180 Gallons	N$1.85
Severe Winter (SW)	230 Gallons	N$2.00
Prolonged Winter (PW)	250 Gallons	N$3.00

Formulate a game model and employ the Minimax criterion technique to determine the gallons of oil Eugene should stockpile at the current price of N$1 per gallon to avoid wasted unused oil and to maximise his saving.   (17 Marks)

QUESTION 2 [25 MARKS]

(a) Two suspects A and B have been apprehended for a crime and are in cells in Tsumeb police station, with no means of communicating with each other. The prosecutor has separately told them the following: (12 marks)

If you confess and agree to testify against the other suspect, who does not confess, the charges against you will be dropped and you will go scot-free. If you do not confess but the other suspect does, you will be convicted and the prosecution will seek the maximum sentence of three years. If both of you confess, you will both be sentenced to two years in prison. If neither of you confesses, you will both be charged with misdemeanors and will be sentenced to one year in prison.

(i) Selecting Suspect A as the row player in a 2-person game, construct the game payoff matrix and each suspect's payoff matrix to determine what the two suspects should do and discuss fully why.

(ii) (ii) Discuss the implication of the dominant strategy for each player (or prisoner).

(b) Consider a competition between two companies, Coca-Cola and Pepsi, and assume the former is thinking of cutting the price of its iconic soda. If it does so, Pepsi may have no choice but to follow suit for its cola to retain its market share. This may result in a significant drop in profits for both companies. Let's assume that the incremental profits that accrue to Coca-Cola and Pepsi are as follows: If both keep prices high, profits for each company increase by $500 million (because of normal growth in demand). If one drops prices (i.e. defects) but the other does not (i.e. cooperates), profits increase by $750 million for the former because of greater market share and are unchanged for the latter. If both companies reduce prices, the increase in soft drink consumption offsets the lower price, and profits for each company increase by $250 million.   (13 marks)

(i)   Considering the above as an example of applications of Prisoner's dilemma problem, construct the payoff matrix for the game model, taking Coca-Cola as the row player and each company's payoff matrix.

(ii)   What should each company do?

QUESTION 3 [25 MARKS]

(a) A licensed mining company in possession of a natural resource field has 0.25 chance for diamond discovery. However, the company has the options to either mine the resource or to sell it the field to another mining company wishing to buy the land for $90,000,000. The cost of mining by the licensee is $100,000,000 with a revenue yield of $800,000,000 if diamond is found. Formulate a game of strategy model to perform a mathematical decision analysis of the problem and determine the decision of the licensee, stating appropriate assumptions for your method.   (8 Marks)

(b) State the Maximum Likelihood Criterion and confirm the above decision obtained in (a) with the criterion.   (4 Marks)

(c) Assuming the company feels that the true chances of discovering diamond are likely to be between 15% and 35%, define sensitivity analysis and the decision crossover point, and hence show that the decision is sensitive to these prior probabilities, providing an appropriate sketch to substantiate your decision analysis and conclusion.   (13 Marks)

QUESTION 4 [20 MARKS]

(a) Discuss the Kendall's classification of Queuing Systems?   (5 Marks)
Explain specifically the M/M/1 queuing system and the process N(t) describing its state at time t as a birth-death process. Provide its state independent parameter equations and define its Traffic Intensity.

(b) Consider a drive-in banking service modelled as an M/M/1 queuing system with customer arrival rate of 2 per minute. It is desired to have fewer than 5 customers line up 99% of the time. How fast should the service rate be?   (4 Marks)

(c) Trucks arrive at garage for a stop-over service according to a Poisson process at a rate of one per every 13 minutes, and the garage service time is an exponential rate variable with mean 9 minutes.

(i) Find the average number L of trucks, the average time W a truck spends in the garage, and the average time W_q a truck spends in waiting for service. (5 Marks)

(ii) Due to increased traffic, suppose that the arrival rate of the trucks increases by 5%. Find the corresponding changes in L, W, and W_q. (5 Marks)

(iii) Discuss your observations. (1 Marks)

END OF EXAMINATION   TOTAL MARKS = 100